A Critic Looks at QBism

Guido Bacciagaluppi

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Abstract

This chapter comments on that by Chris Fuchs on qBism. It presents some mild criticisms of this view, some based on the EPR and Wigner's friend scenarios, and some based on the quantum theory of measurement. A few alternative suggestions for implementing a subjectivist interpretation of probability in quantum mechanics conclude the chapter.

"M. Braque est un jeune homme fort audacieux. [...] Il méprise la forme, réduit tout, sites et figures et maisons, à des schémas géométriques, à des cubes. Ne le raillons point, puisqu'il est de bonne foi. Et attendons."[1] Thus commented the French art critic Louis Vauxcelles on Braque's first one-man show in November 1908, thereby giving cubism its name. Substituting spheres and tetrahedra for cubes might be more appropriate if one wishes to apply the characterisation to qBism — the view of quantum mechanics and the quantum state developed by Chris Fuchs and co-workers (for a general reference see either the paper in this volume, or Fuchs (2010)). In this note, I shall not comment on other possible analogies, nor shall I present an exhaustive critical review of qBism (for an excellent one, see Timpson

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[1] "Mr Braque is a very bold young man. [...] He despises form, reduces everything, places and figures and houses, to geometric schemes, to cubes. Let us not rail him, since he is in good faith. And let us wait." (My translation from Vauxcelles (1908).)

(2008)). I simply wish to air a couple of worries I think qBists ought to think about more, in a friendlier spirit than the one Braque and Picasso's paintings might have encountered one hundred years ago.

As was mentioned in discussion, qBism ought perhaps to stand not for 'quantum Bayesianism', for there are many kinds of Bayesians who would not recognise themselves in it, but for 'quantum Brunism', after Bruno de Finetti, who championed the radical subjectivist or pragmatist view of probabilities. In Section 1, thus, I shall briefly remind the reader just how radical this view is. In Section 2 I shall then express a number of worries about the quantum case. Finally, Section 3 will give a quick sketch of some alternative pragmatist options for interpreting quantum probabilities.[2]

1 Brunism, classical and quantum

First of all, what are subjective probabilites (aka credences), or what are they *for* in a pragmatist understanding of probability? They are strategies we adopt in order to navigate the world. Some may turn out to be more useful than others in practice, but that does not change the fact that they are subjective constructs.

Take for instance (classical) coin tossing. Subjective probabilities describe our expectations for outcomes of succesive tosses (and guide our betting behaviour). We start off with certain priors; these are updated as we go along, reflecting past performance. But where do our original priors come from? Let us dispel the idea that they stem from some intuitive feel or mystic vision. They might, but they generally will not, and indeed need not. Especially in more complex situations, we can and will adopt a theoretical model to choose our priors. Such a model may involve a thorough analysis of the whole set-up, or be even quite crude and just make use of simple properties of the coin alone (e. g. weight distribution). In any case, it will involve consideration of *objective* (but non-probabilistic!) properties of the system under consideration.

[2]With the addition of Subsection 2.1, the material in this note was presented at the Bertinoro conference. My thanks go to Maria Carla Galavotti, Roberta Campaner, Beatrice Collina and all the colleagues in the ESF network who organised the conference. But most of all I wish to thank Chris Fuchs for the pleasure of endless discussions of quantum Bayesianism over the years, and for removing the prejudice in my mind that a subjective interpretation of quantum probabilities must be a non-starter.

If the observed frequencies match the ones determined through the model, it may be that the data 'confirm' our theoretical model. This, however, does not mean that our strategy is any less subjective. Indeed, there is no *necessary connection* between, say, weight distribution and relative frequencies: for any possible sequence of results there is an initial condition that will produce it, irrespective of the details of the weight distribution of the coin. Nor is there for the 'Brunist' any *compelling rational justification* in the sense of David Lewis (1971) for letting our credences depend on the weight distribution, only pragmatic criteria such as simplicity, past performance etc. 'Confirmation' is only a reflection of such past performance, and bears no guarantee of future reliability.

One might object that de Finetti himself showed that if one's priors for sequences of results are exchangeable, the posteriors will converge (with subjective probability 1). This is surely a sign of tracking something 'out there' (or so the objection goes).[3] As long as we are dealing with finite samples, however, any apparent convergence of one's probability assignments (one's own or intersubjectively) again merely reflects past performance. And the expectation of future convergence depends crucially on the subjective assumption of exchangeability. Exchangeability is an assumption about the structure of one's subjective priors, and is entirely independent of any objective behaviour of the system under consideration.

Even this more technical objection, thus, does not alter the fact that according to de Finetti there is no sense in which our probability judgements are *right or wrong*. As de Finetti very graphically expresses,

PROBABILITIES DO NOT EXIST.

The same is true in qBism. Quantum probabilities according to qBism simply *are* subjective probabilities in the sense of de Finetti. They are strategies that we adopt in navigating an (admittedly) unexpectedly strange world. They may be impressively successful in terms of calculational power, past performance etc., but that makes them no less subjective than the theoretical models we might adopt for coin tossing (or indeed in classical statistical mechanics!).

What may strike one as strange, or at least unfamiliar, is applying de

[3]Such a view seems to be suggested for instance by Greaves and Myrvold (2010).

Finetti's ideas to a case which is generally thought to be genuinely *indeterministic*. There may be a price to pay if one does so, because if probabilities are not objective, then any use of them in describing the observed regularities in the world cannot be thought of as expressing law-like behaviour in any *necessary* sense. But if one is happy with a broadly Humean view of laws, this will not strike one as a disadvantage of the application of de Finetti's views to an indeterministic context.

Another point worth making explicitly (because it is not normally included in presentations of qBism), is that even in the quantum case one may adopt theoretical models for fixing one's priors. For instance, one might use techniques for selecting a Hamiltonian and calculating its ground state, and set one's subjective probabilities according to that. The data might then seem to 'confirm' some particular choice of Hamiltonian. But according to the qBist there will be no necessity or rationally compelling reasons for using it to fix our credences, only pragmatic ones such as calculational power, past performance etc.

Fuchs generally talks of quantum states themselves as subjective (and this leads him also to a view of Hamiltonians as subjective). What I am saying instead is that one can take Hamiltonians to be objective properties of quantum systems, just like the weight distribution in a coin, and further as *non-probabilistic* properties, again just like weight distribution. Indeed, one can even think of quantum states (!) as objective properties of a system, stripping them of their customary probabilistic elements, and thinking for instance of the ground state 'merely' as a state of definite energy. (After all, Heisenberg, Pauli, Schrödinger and others all had notions of quantum states *before* Born introduced his 'statistical' interpretation of the wave function.[4]) The probabilistic association would attach to such a state exclusively in the way we use it to fix our credences. We shall return to the idea of objective non-probabilistic quantum states in Section 3.

[4]When Schrödinger introduced his wave functions, he clearly understood them as representing physical states of matter. But also Heisenberg and other advocates of matrix mechanics had a notion of stationary state, both preceding and distinct from Schrödinger's wave functions. Cf. Bacciagaluppi and Valentini (2009, Ch. 3) and Bacciagaluppi (2008).

2 Worries

2.1 EPR and Wigner's friend

One worry I wish to air about qBism relates to one of its reputed major selling points, namely its 'local' account of 'nonocal' EPR correlations. The qBist story goes like this. Consider Alice's credences about *Bob's* electron. Her initial credences might be equal to 1/2 for the result of any spin experiment she might perform on Bob's side. But if she performs first a spin measurement on her own side — say in direction x with result 'up' —, she will then update her credences about further measurements on Bob's electron. In particular she will then believe with all her heart (credence 1) that if she performs a spin-x experiment on Bob's electron, this will give the result 'down'. What has changed, however, are simply *her own credences.* Similarly, Bob has credences about Alice's electron, which also change in an analogous manner, but although they are *about* Alice's electron, they are *his* credences, and (insofar as beliefs can be said to be located anywhere[5]) they are in his head. These two autonomous points of view can be then married together to form a composite picture of the EPR pair. (Thus, indeed, bringing out an analogy between qBism and cubism!)

The worry is about this marriage. QBism, just like 'classical Brunism', presupposes that different agents be able to share data, an assumption that underlies the intersubjective agreement between different agents derivable from de Finetti's theorem. Different agents may become aware at different times of different pieces of data, but they can inspect each others' data and pool them together.

Indeed, suppose Alice and Bob both perform spin measurements in direction x, and then later meet to compare results. From Alice's point of view, her asking Bob his result is her own measurement of Bob's electron (which in the meantime has interacted with this further system called Bob), and her own measurement results are timelike related. But if she wishes to take Bob's own report seriously as providing her with data that at the time of his measurement were available to him but not yet to her, then the mystery of perfectly correlated spacelike separated events returns.[6]

[5]Disregarding notions of extended cognition — which are presumably beside the point here.

[6]The situation is quite analogous to that of collapse on the forward light cone. If tech-

Wigner's friend (another case for which qBism claims to have a ready explanation) can be seen as a variant of the above. Bob performs an experiment in a lab that is isolated from Alice (perhaps because Alice and Bob are spacelike separated at the time). Alice can later perform a measurement on the content of Bob's lab (either repeating Bob's measurement or asking him for a report). If she does repeat Bob's measurement, the result she observes coincides with the result of the earlier measurement as reported by Bob, again suggesting that she should take the report seriously as describing objective data that were not yet available to her. Unlike the EPR case, this is not particularly puzzling. But in the Wigner's friend scenario, we are invited to consider also the case in which Alice performs instead an interference experiment on the entire contents of Bob's lab, and thereby 'quantum erases' Bob's result. In qBist terms, this could be understood merely as Alice performing some manipulation that leads her to change her own credences about the results of her asking Bob what he has seen. She now expects from Bob not some or other report of a definite result, but a definite report of not having performed the experiment. But this description misses out on the fact that Alice's manipulation has in fact obliterated also Bob's piece of data and any memory that Bob had of it (unless, that is, we assume that Bob did not really possess any such piece of data in the first place).

Thus, if we believe that data obtained by different agents are equally objective, thus understanding 'pooling of data' literally, we have problems. There are no such problems once the data have been pooled together, but we have two puzzling cases in situations where Bob's data are not yet available to Alice. In the EPR case, qBism remains silent on why Alice and Bob's data should be correlated, and in the Wigner's friend case, it remains silent on how Alice can erase Bob's data. The choice for qBists seems to be between: (a) providing us with a further story about data and/or agents *themselves*, rather than just strategies for how agents update their credences in the face of new data; and (b) some kind of solipsism or radical relativism, in which we care only about single individuals' credences, and not about whether and how they ought to mesh.

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nically feasible, a theory in which the collapse of the quantum state takes place along the forward light cone of a triggering event would be manifestly Lorentz-invariant. However, in the case of an EPR pair, it would leave unexplained how space-like separated collapses are correlated so as to match up on the overlap of their future light cones. Cf. e.g. the discussion in Bacciagaluppi (2010).

2.2 Hidden constraints on probability assignments

My main worry, however, runs deeper in the conceptual foundations of qBism. A central idea of qBism is that the view is not a modification of but an *addition to* Bayesian coherence. This addition is equally normative, but rather than being rational in origin, it is empirically motivated. It is essentially contained in the formula

$$Q(D_j) = (d+1)\sum_{i=1}^{d^2} P(H_i)P(D_j|H_i) - 1 \ , \tag{1}$$

which constrains the relation between probabilities in certain pairs of actual and counterfactual situations. The actual situation is a measurement of the family of projections D_j, and the formula compares the probabilities $Q(D_j)$ with the probabilities $\sum_{i=1}^{d^2} P(H_i)P(D_j|H_i)$ that *would* have been obtained if a previous measurement of the 'fiducial SIC' with effects H_i had been performed (a generalisation of this formula holds if the D_j are effects).[7] This is, indeed, a situation on which Bayesian coherence is silent. The law of total probability

$$Q(D_j) = \sum_{i=1}^{d^2} P(H_i)P(D_j|H_i) \tag{2}$$

is clearly a prescription for relating the probabilities of two *actual* measurements.

However, I claim that formula (1) already presupposes very strong constraints on our subjective probability assignments. Indeed, the very idea of well-defined probabilities $Q(D_j)$ for projections (or more generally effects) D_j already embodies such strong constraints, even before we start comparing our probability assignments for actual measurements to our probability assignments for counterfactual measurements.

In order to substantiate this claim, let me recall some standard material. It is nowadays customary in quantum mechanics and quantum information to describe (general) measurements using the concepts of *operations* and of *POVM*s (positive-operator-valued measures).

[7] See below for the definition of an effect.

Operations are families of transformations on the quantum states (thought of as transformations induced by 'measurements'). Such transformations could for instance be of the form ('pure operation')

$$\rho \mapsto A_i \rho A_i^* \tag{3}$$

(with the right-hand side suitably renormalised), and each such transformation takes place with probability

$$\mathrm{Tr}(A_i \rho A_i^*) = \mathrm{Tr}(\rho A_i^* A_i) \ . \tag{4}$$

For this expression to indeed define a probability distribution over the various possible transformations we must have:

$$\sum_i D_i := \sum_i A_i^* A_i = \mathbf{1} \tag{5}$$

(with $\mathbf{1}$ the identity operator). The thus defined operators D_i are so-called *effects*, i. e. they are positive (self-adjoint with positive spectrum) and with spectrum contained in the interval $[0, 1]$. The mapping from the indices i (or sets thereof) to the associated D_i (or sums thereof) is thus an effect-valued measure, also called positive-operator-valued measure (POVM).

One should note crucially that the probabilities associated with a transformation are fixed just by the corresponding POVM.

Further, one easily sees that a pure operation such as the above can always be implemented by coupling the system to an ancillary system,

$$|\psi\rangle \otimes |\varphi_0\rangle \mapsto \sum_i A_i |\psi\rangle \otimes |\varphi_i\rangle \tag{6}$$

for some orthonormal family $|\varphi_i\rangle$, and then collapsing onto the latter. Note that such coupling is indeed unitary because of (5). (This result, suitably generalised to all operations, is known as the Naimark dilation theorem.)

Here is a very familiar example.

Example 1 (von Neumann measurement):
Let $A_i := |\psi_i\rangle\langle\psi_i|$ for some orthonormal basis. We implement it via

$$\sum_i \alpha_i |\psi_i\rangle \otimes |\varphi_0\rangle \mapsto \sum_i \alpha_i |\psi_i\rangle \otimes |\varphi_i\rangle \ , \tag{7}$$

and we have

$$A_i^* A_i = P_i := |\psi_i\rangle\langle\psi_i| \ , \tag{8}$$

so the correspondiong POVM is projection-valued.

A von Neumann measurement, however, is not the only experimental procedure for measuring a projection-valued measure, as the following example shows.

Example 2 ('measurement of the second kind'):
Let the $|\psi_i\rangle$ be as above, and let $B_i = |\psi_i'\rangle\langle\psi_i|$ for some arbitrary unit vectors $|\psi_i'\rangle$. We have

$$\sum_i \alpha_i |\psi_i\rangle \otimes |\varphi_0\rangle \mapsto \sum_i \alpha_i |\psi_i'\rangle \otimes |\varphi_i\rangle \ . \tag{9}$$

Note that

$$B_i^* B_i = |\psi_i\rangle\langle\psi_i'|\psi_i'\rangle\langle\psi_i| = P_i \ , \tag{10}$$

and the transformation is indeed associated to the *same* projection-valued measure as in Example 1. The difference is that the 'collapsed' state of the system after the measurement is no longer an eigenstate of the measured observable (in the traditional sense of a self-adjoint operator with spectral measure defined by the P_i). The measurement is not 'minimally distrurbing'.

How do these examples lead to a worry about qBism? Note that (1) contains probability assignments $Q(D_j)$ referring to measurements of POVMs *irrespective* of which transformations are used to implement them. Thus, it presupposes that we assign the same probabilities to the results of the two transformations in Examples 1 and 2, even though they correspond to *different lab procedures*. QBism is currently silent on why we have this constraint on our probability assignments. Of course, we can say it is empirically well-established, and we can derive it theoretically from the quantum mechanical theory of measurement *if* we apply the usual Born rule to the ancillary system in the model. But if (1) is meant to be a simple axiom embodying one of the main modifications of Bayesian coherence theory that are supposed to *lead* to quantum mechanics, it appears that it already presupposes a lot of the structure it is trying to explain.

The point can be made even more strikingly using a further example.

Example 3 (sequential von Neumann measurements):
Concatenating two operations yields a further operation, e. g.

$$\rho \mapsto P_i \rho P_i \mapsto Q_j P_i \rho P_i Q_j \ . \tag{11}$$

Indeed, defining

$$A_{ij} := Q_j P_i \ , \tag{12}$$

we obtain a corresponding POVM:

$$\sum_{ij} A^*_{ij} A_{ij} = \sum_{ij} P_i Q_j P_i = \sum_i P_i = \mathbf{1} \ . \tag{13}$$

This POVM can be measured via two sequential von Neumann measurements. For instance, if the P_i and Q_j project onto spin-1/2 eigenstates in directions x and y, we can implement the composite POVM by letting an electron pass in sequence two Stern–Gerlach magnets at right angles to each other (two sequential interactions between the spin and spatial degrees of freedom of the electron) and then measuring on which *quadrant* of the screen the electron impinges.

But we can also implement it using a *single* interaction with an ancillary system, e. g. by defining

$$B_{ij} := \sqrt{P_i Q_j P_i} \ . \tag{14}$$

Indeed,

$$B^*_{ij} B_{ij} = (\sqrt{P_i Q_j P_i})^2 = P_i Q_j P_i = A^*_{ij} A_{ij} \ , \tag{15}$$

and the corresponding POVM is the same as in (13).

This is again a case of two totally different laboratory procedures that allow one to measure the same POVM, this time an effect-valued rather than projection-valued one.

We can again note that the qBist formula relating the probabilities we assign to results of (actual) measurements of effect-valued measures to the probabilities in the corresponding counterfactual situations (a fairly straightforward generalisation of (1)) presupposes that we assign the same probabilities also to these two procedures.

This presupposition, however, is even more suspect than in the case of Examples 1 and 2, because the procedure defined by (12) already implicitly contains the *collapse* of the state (or at least the effective collapse through decoherence of the spin state by the spatial degree of freedom along the first measured direction x), which even more strongly suggests that the strategy currently pursued within qBism for axiomatising and/or understanding quantum mechanics implicitly presupposes a lot of the structure it is trying to explain.

3 Other pragmatist alternatives

Whether or not qBism will turn out to provide a fully successful new framework for understanding quantum mechanics, it has already shattered a taboo: that of using a subjectivist approach to probabilities in quantum mechanics. The traditional view of course is that quantum probabilities are *the* paradigm of objective probabilities. The idea that a radical subjectivist approach *à la* de Finetti might be applied to the quantum case (which at least *prima facie* is truly indeterministic) used to be inconceivable.

The inconceivable having now been conceived, I wish to suggest in this section that in fact a subjectivist/pragmatist approach to probability can be applied in the context of just about *any* approach to quantum mechanics. In particular, it can be applied also to approaches that adopt an ontic view of the quantum state. As pointed out in Section 1, an ontic view of the quantum state might be adopted also within qBism, as long as the quantum state itself is not seen as a probabilistic entity (although this is not part of the usual presentations of qBism[8]). Other approaches to the foundations of quantum mechanics such as the Bohm theory, GRW theories, or the Everett theory explicitly take an ontic view of the quantum state (thus at least in principle also providing an answer to the question of the ontology of data and agents, cf. above Section 2.1).

I shall now sketch very briefly in turn how each of these can adopt a radical subjectivist view of probabilities, thus taking the quantum state as ontic but as non-probabilistic (whether or not it is customary for them to do so!).

3.1 Bohm

It is easiest to think of subjectivism about probabilities in the case of the Bohm theory. Indeed, the Bohm theory is a deterministic theory, and we are familiar with applying the classic de Finetti analysis to deterministic cases.

[8] I cannot resist teasing Chris here by pointing out that *pace* the remarks in Fuchs (2010, pp. 24–25), the results of decoherence just scream out for an ontic interpretation of the quantum state (so much classical-like structure within the quantum state that could be used to explain the classical world — *if only* we could avail ourselves to an ontic interpretation of the state). Cf. Bacciagaluppi (2012, esp. Section 3.5). The quantum-to-classical transition is here to stay, and qBism ought to incorporate it.

The Bohm theory (or de Broglie–Bohm theory, or pilot-wave theory) describes 'classical' configurations evolving deterministically in a way fixed by the quantum wave function of the total system. The usual statistical predictions of quantum mechanics are recovered if one assumes that the configurations in an ensemble are distributed according to the usual quantum mechanical distribution (a condition which is preserved over time). The situation is very similar to that of classical statistical mechanics (with the difference that now the 'equilibrium' distributions are time-dependent), and this analogy has been developed in considerable detail.[9]

Note that the quantum states in the Bohm theory are both ontic *and* non-probabilistic. They are the 'pilot waves' of the theory. They acquire a probabilistic significance only if we adopt a strategy of choosing our subjective probabilities to be equal to the modulus squared of the wave function. While such a strategy is highly successful,[10] the quantum wave functions provide *no fundamental constraint* on initial positions, nor indeed on particle distributions in ensembles. This is evident from the fact that *non-equilibrium* pilot-wave theory is equally intelligible and may even have very interesting applications (Valentini 2010).

Just as in the case of the weight distribution of a coin — which turns out to be a reliable indicator of the statistical behaviour under repeated tossing only under certain 'typical' conditions —, so the wave function describing an ensemble of particles (more precisely, the so-called effective wave function, i. e. the component of the wave function responsible for the motion) is a reliable indicator of the statistical behaviour of the particles only on the assumption of typicality. Thus in both cases a physical but non-probabilistic property associated with the system under consideration (and one that can be modelled theoretically) is used as a pragmatic short-cut for fixing our subjective probabilities for the behaviour of the system.

[9] For general references to the Bohm theory, see e. g. Bohm and Hiley (1993), Goldstein (2013), Holland (1993).

[10] And can be justified using arguments analogous to those employed in classical statistical mechanics (see e. g. Valentini (1991a,b), Dürr, Goldstein and Zanghì (1992), Towler, Russell and Valentini (2011)).

3.2 GRW

It is less familiar to think in terms of subjective probabilities in the case of spontaneous collapse theories, i. e. theories in which the Schrödinger evolution is modified in a way that reproduces the phenomenology of collapse. Such theories were shown to be viable by Ghirardi, Rimini and Weber (1986) and Pearle (1976, 1989), and are thus also know as GRW (or GRWP) theories.[11]

Since in GRW theories we have genuine indeterministic evolution of the quantum state, they are generally thought of in terms of objective collapse probabilities, much like 'traditional' quantum mechanics. However, the idea that in the case of genuine indeterminism probabilities should be thought of as objective presupposes that a viable account of objective probabilities be given, e. g. in terms of frequencies or propensities, and both of these accounts suffer from more or less severe problems. The third strategy open to objectivists is to apply Lewis's (1971) 'principal principle', i. e. to argue that there are compelling rational reasons for adopting a particular recipe to fix one's subjective probabilities. Perhaps some version of 'Humean objective chances' can deliver on this, but the step to subjectivism might be very short in that case.[12]

The quantum state in GRW theories is clearly ontic, indeed at least *prima facie* provides the 'stuff' the world is made of.[13] But as regards the probabilistic *evolution* of the state, we can adopt de Finetti's position, holding that there are no right or wrong probabilities about how the state evolves. We can take the GRW theory (a theoretical model of these probabilities) as a pragmatic recipe for fixing our *subjective* probabilities for the dynamical behaviour of the states (much as we take weight distributions to guide our expectations about the behaviour of tossed coins). And we can push the line that de Finetti's position is not only tenable in an indeterministic context, but that it may be even less artificial than others in the context of GRW theories.

[11] For an accessible reference, see Ghirardi (2011).

[12] On Humean objective chances, see e. g. Hoefer (2007), and for their application to GRW and for more general discussion of objective probabilities in GRW, see Frigg and Hoefer (2007).

[13] There is a debate about the most natural interpretation of spontaneous collapse theories: whether — in increasing order of resilience against the so-called 'tails' problem — it is in terms of wave functions, matter density, or collapse events (so-called 'flashes' or 'hits'). For details see e. g. Ghirardi (2000), Allori *et al.* (2008), Bacciagaluppi (2010).

3.3 Everett

The final and most interesting case is that of Everett.[14] As is well known, Everett takes the wave function of the universe seriously as providing the ontology of the theory, and the Schrödinger equation as providing its dynamics. Collapse is explained through the correlational structure of the universal wave function, whereby the quantum state *appears* to collapse to an internal observer (or whatever other system is 'recording' collapse events). Each outcome of a collapse is equally real, *relative* to the corresponding component of the observer.

Modern-day Everettians refine Everett's original analysis of the correlational structure of the universal wave function through the use of decoherence theory (cf. e. g. Wallace (2010a), Bacciagaluppi (2012)). However, there is a generalised perception of a problem in making sense of probabilities in the Everett theory, precisely because, say, in the context of a sequence of measurements on an ensemble of systems, *all* sequences of outcomes are actualised (in different 'worlds' or 'branches' of the universal wave function). Thus, not only are there no right or wrong probabilities, but probabilities would appear to make no sense at all (at least in the sense that we do not seem to be ignorant of what will be the outcome of a measurement[15]).

A breakthrough on this question was achieved not many years ago by David Deutsch (1999) and David Wallace (2007, 2010b), who adopted the Lewisian strategy sketched above and argued, first, that rational decision theory can be applied to the case of an Everettian agent located before a branching event (see also Greaves (2004) and Greaves and Myrvold (2010)), and, crucially, that *rationality constraints* on such an agent will force them to adopt the quantum probabilities for the results of the branching. Thus, quantum probabilities (at least insofar as they apply to such a decision-theoretic situation) are objective chances in the sense of Lewis. The approach based on the Deutsch–Wallace theorem appears to command quite a consensus among

[14]Cf. also my comments in Bacciagaluppi (2013), which reviews the state-of-the-art volume on the Everett theory edited by Saunders *et al.* (2010). Everett's complete writings on quantum mechanics, together with a wealth of other original material, have been published and annotated by Barrett and Byrne (2012).

[15]Note that, as clearly shown by Vaidman (1996), we do have ignorance of the result of a measurement at least *after* the measurement has occurred and we are not yet aware of the result. After the branching induced by a measurement there is a genuine question about self-location.

modern Everettians, but has also been the object of strong criticism.[16]

I wish to suggest that Everettians can avail themselves of an alternative *subjectivist* strategy, taking 'branch weights' as guides for navigating a branching universe. The choice of branch weight as quantifying probability (or 'typicality') can be pragmatically justified on the basis that it has performed well in the past, and on the basis of its being the 'natural' measure on branches, e. g. because of Gleason's (1957) theorem or the Deutsch–Wallace theorem (this is Greaves and Myrvold's (2010) take on the latter), or because it is conserved by the dynamics analogously to the measure in classical statistical mechanics (this is in fact the justification proposed by Everett (see e. g. Barrett and Byrne 2012, pp. 274–275)). This reading of probability in Everett needs of course to be developed further, but would provide a particularly striking way of combining an ontic view of the quantum state with a subjectivist view of quantum probability.[17]

Such an application of 'Brunism' to the quantum case would of course be much tamer than Chris Fuchs's usual emphasis on quantum states themselves being subjective. Stretching the metaphor, if we allow Everett's universal wave function to explain what agents and outcomes are in the first place, the heroic phase of 'analytical qBism' will give way to a much tamer 'synthetic qBism'. That said — returning to the history of art — I have always preferred the analytic phase of *cubism* to the synthetic one!

[16]For a lively and representative sample of the literature, see the relevant contributions by Albert, Greaves and Myrvold, Kent, Price, Saunders, and Wallace, as well as the transcripts of the discussions, in Saunders *et al.* (2010). In particular, Price (2010) argues that agents may have *global* reasons on which to base their decisions, i. e. reasons other than the utilities of their successors. Such arguments of course undermine the idea that there should be compelling rational arguments for adopting the usual quantum probabilities in Everett.

[17]In further work, in particular with my graduate students, I hope to elaborate both on the analogy between Everett's view of probability in his own theory and in classical statistical mechanics — in particular on how it allows one to make statistical inferences in an Everettian universe —, and on the pragmatist reading of typicality, both in Everett and in classical statistical mechanics.

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